Apr. 5, 2017

Sect. 11-3

Geometric Series & Sequences

Defn

Finding Pattern

Finding n^{th} term

Geometric Mean

Finding Sum

Sigma Notation

2, 4, 8, 16, 32, 64, 128

×2 ×2 ×2

This is a geometric sequence because the same # is multiplied to get the next term.

Common ratio: $r = 2$

In General

$$r = \frac{a_n}{a_{n-1}}$$

$2, 4, 8, \ldots$ Find a_{10}

$$a_n = a_1 \cdot r^{n-1}$$

$$a_{10} = 2 \cdot (2)^{10-1} = 2 \cdot 2^9$$

$$= 2 \cdot 512$$

$$a_{10} = 1024$$

$2, 6, 18, \underline{54}, \underline{162}, \underline{486}$

$r = \frac{6}{2} = 3$

Find a_8

$a_8 = 2 \cdot (3)^{8-1} = 2 \cdot 3^7$

$a_8 = 4374$

$3, -6, 12, -24, \underline{48}, \underline{-96}, \underline{192}$

$r = -2$

$a_9 = 3 \cdot (-2)^{9-1} = 3 \cdot (-2)^8$

$= 3 \cdot 256$

$= 768$

$9, 3, 1, \frac{1}{3}, \frac{1}{9}, \frac{1}{27}$

"Divide by 3"

$r = \frac{3}{9} = \frac{1}{3}$

$a_7 = 9 \cdot \left(\frac{1}{3}\right)^{7-1} = 9 \cdot \left(\frac{1}{3}\right)^6$

$= \frac{9}{1} \cdot \frac{1}{729} = \frac{1}{81}$

Geometric Mean

$3, \underline{\quad}, 9, \underline{\quad}, 27$

$\times r \quad \times r$

$r^2 = \frac{27}{3}$

$r = \sqrt{9}$

$r = 3$

$2, \underline{\quad}, 4, \underline{\quad}, 8, \underline{\quad}, 16$

$\times r \quad \times r \quad \times r$

$r^3 = \frac{16}{2}$

$r = \sqrt[3]{8}$

$r = 2$

Finding the Sum

$2 + 4 + 8 + \ldots\ldots + a_n$ (Stop)

$$S_n = \frac{a_1(1-r^n)}{1-r}$$

or *

$$S_n = \frac{a_1 - ra_n}{1-r}$$

*Depending on info given

$2+4+8+\ldots\ldots$ 6 terms

$r=2$

$$S_6=\frac{2(1-2^6)}{1-2}$$

$$=\frac{2(1-64)}{-1}=\frac{2(+63)}{+1}$$

$$S_6=126$$

S_n for $a_1 = 3$ $a_n = 81$ $r = 3$

$$S_n = \frac{3 - 3(81)}{1 - 3}$$

$$= \frac{3 - 243}{-2}$$

$$= \frac{-240}{-2} \qquad S_n = 120$$

$9+3+1+\ldots \quad r=\frac{1}{3}$ Find S_5

$$S_5=\frac{9\left(1-\left(\frac{1}{3}\right)^5\right)}{1-\frac{1}{3}}=\frac{9\left(1-\frac{1}{243}\right)}{\frac{2}{3}}$$

$$=\frac{9\left(\frac{242}{243}\right)}{\frac{2}{3}}=\frac{\frac{242}{27}}{\frac{2}{3}}=\frac{242}{27}\cdot\frac{3}{2}$$

$$S_5=\frac{121}{9}$$

Infinite Sum

$9+3+1+\frac{1}{3}+\frac{1}{9}+\frac{1}{27}+\frac{1}{81}+\frac{1}{243}+\frac{1}{729}+\ldots\infty$

$$S_\infty = \frac{a_1}{1-r}$$

$$S_\infty = \frac{9}{1-\frac{1}{3}} = \frac{9}{\frac{2}{3}}$$

$$= \frac{9}{1}\cdot\frac{3}{2}$$

$$S_\infty = \frac{27}{2}$$

Sigma Notation

$$\sum_{i=1}^{4} \left(\frac{2}{3}\right)^i$$

Expand: $\frac{2}{3} + \frac{4}{9} + \frac{8}{27} + \frac{16}{81} = \frac{130}{81}$

$$S_n = \frac{\frac{2}{3}\left(1 - \left(\frac{2}{3}\right)^4\right)}{1 - \frac{2}{3}} = \frac{130}{81}$$

$$\sum_{i=1}^{\infty} \left(\frac{2}{3}\right)^i$$

$$S_\infty = \frac{\frac{2}{3}}{1-\frac{2}{3}}$$

$$= \frac{\frac{2}{3}}{\frac{1}{3}} = \frac{2}{3} \cdot \frac{3}{1}$$

$$S_\infty = 2$$

Non-Arith. or Geo. Sigma Props.

$$\sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6}$$

$$\sum_{i=1}^{n} i^3 = \frac{n^2(n+1)^2}{4}$$

$$\sum_{i=1}^{14}(2i^2-3) = 2\sum_{i=1}^{14} i^2 - \sum_{i=1}^{14} 3$$

$$= 2\left[\frac{14(14+1)(2(14)+1)}{6}\right] - 14(3)$$

$$= \frac{14(15)(29)}{3} - 42$$

$$= 14(5)(29) - 42$$

$$= 2030 - 42$$

$$= 1988$$